Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. Two particles, P and Q, are moving towards each other in opposite directions along the same straight line when they collide directly. Immediately before the collision the speed of Q is $2u$. The mass of Q is $3m$ and the magnitude of the impulse exerted by P on Q in the collision is $4mu$.

Find

(a) the speed of Q immediately after the collision,

(3)

(b) the direction of motion of Q immediately after the collision.

(1)

a) Draw a diagram of the masses and velocities.

The formula for Impulse is : $\boxed{I = mv - mu}$ for particle Q.

$\therefore \; 4mu = 3m(v) - 3m(-2u)$

$\therefore \; 4mu = 3mv + 6mu$

$3mv = -2mu$

$v = \dfrac{-2mu}{3m} = \dfrac{-2u}{3}$ Since speed can't be negative, $v = \dfrac{2u}{3}$ //

b) Since we calculated v to be negative when positive is the opposite direction to the original motion means it continues to travel in its original direction.

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Question 1 continued

Q1

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2. A motorbike is moving with constant acceleration along a straight horizontal road.

The motorbike passes a point P and 10 seconds later passes a point Q.

The speed of the motorbike as it passes Q is $28\,\text{m s}^{-1}$

Given that $PQ = 220\,\text{m}$,

(a) find the acceleration of the motorbike,

(3)

(b) find the distance travelled by the motorbike during the fifth second after passing P

(4)

a) It's stated that acceleration is constant therefore SUVAT can be used.

s : 220 SUVAT formula without initial velocity : $\boxed{s = vt - \frac{1}{2}at^2}$

u :

v : 28 $\therefore 220 = 28(10) - \frac{1}{2}a(10)^2$

a : a $\therefore 220 = 280 - 50a$

t : 10 $\therefore 50a = 60 \quad a = \frac{60}{50} = 1.2\,\text{ms}^{-2}$

b) For part b) we need to find the distance travelled at times t=5 and t=6 to find the distance travelled during the fifth second. Setting up SUVATs for each time :

t=5 s : S_5 Using : $\boxed{s = vt - \frac{1}{2}at^2}$

u :

v : 28 $\therefore S_5 = 28(5) - \frac{1}{2}(1.2)(5)^2 = 125\,\text{m}$

a : 1.2

t : 5

t=6 s : S_6 Using : $\boxed{s = vt - \frac{1}{2}at^2}$

u :

v : 28

a : 1.2 $\therefore S_6 = 28(6) - \frac{1}{2}(1.2)(6)^2 = 146.6\,\text{m}$

t : 6

$\therefore$ Distance travelled during fifth second $= S_6 - S_5 = 146.6 - 125 = 21.4\,\text{m}$

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Question 2 continued

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Question 2 continued

Question 2 continued

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Q2

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3. A tractor of mass 6 tonnes is dragging a large block of mass 2 tonnes along rough horizontal ground. The cable connecting the tractor to the block is horizontal and parallel to the direction of motion.

6000kg 2000kg

The cable is modelled as being light and inextensible.

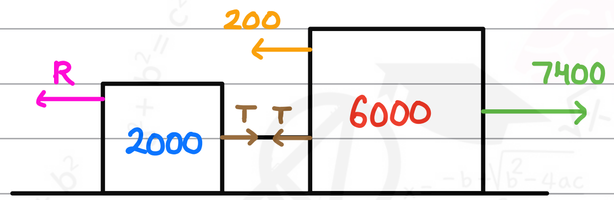
The driving force of the tractor is 7400 N and the resistance to the motion of the tractor is 200 N. The resistance to the motion of the block is R newtons, where R is a constant.

Given that the tension in the cable is 6000 N and the tractor is accelerating,

(a) find the value of R.
(6)

(b) State how you have used the fact that the cable is modelled as being inextensible.
(1)

a) Draw a diagram labelling masses and forces.



Equation for force:

$\boxed{\Sigma F = ma}$

Using the formula above taking positive as rightwards as this is the direction it's accelerating.

For the whole system:

$7400 + T - T - 200 - R = (6000 + 2000)a$

$\therefore 7200 - R = 8000a$

$\therefore R = 7200 - 8000a$

$= 7200 - 8000(0.2)$

$= 5600\text{ N}$

For only the tractor:

$7400 - T - 200 = 6000a$

$\therefore 1200 = 6000a$

$\therefore a = \frac{1200}{6000} = 0.2$

b) Since the cable is modelled as inextendible, the acceleration for the tractor and block is equal.

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Question 3 continued

Q3

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4.

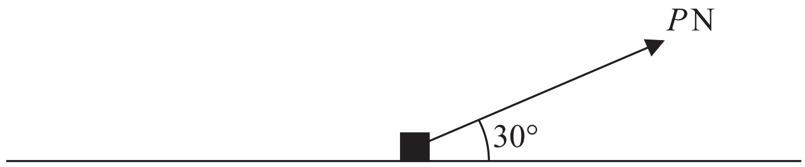


Figure 1

A small block of mass 5 kg lies at rest on a rough horizontal plane.

The coefficient of friction between the block and the plane is $\frac{3}{7}$

A force of magnitude P newtons is applied to the block in a direction which makes an angle of 30° with the plane, as shown in Figure 1.

The block is modelled as a particle.

Given that $P = 14$

(a) find the magnitude of the frictional force exerted on the block by the plane and describe what happens to the block, justifying your answer.

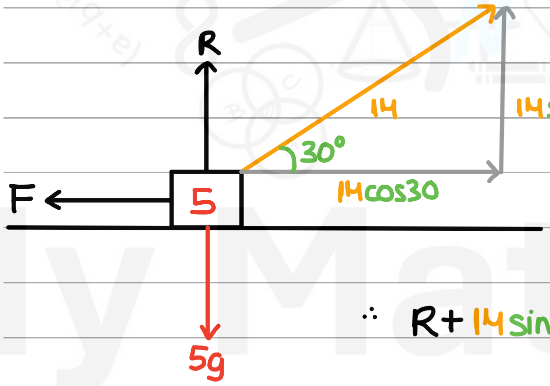
(6)

The value of P is now changed so that the block is on the point of slipping along the plane.

(b) Find the value of P

(6)

a) Draw a diagram labelling and resolving forces.



Since the block is in vertical equilibrium :

$$\boxed{\Sigma F = 0}$$

$\therefore R + 14\sin 30 - 5g = 0 \quad \therefore R = 5g - 14\sin 30 = 42\text{N}$

Using the formula : $\boxed{F = \mu R}$, $F_{max} = \frac{3}{7} \times 42 = 18\text{ N}$

Now we need the horizontal component of P and if its value is greater than F_{max} then $\Sigma F > 0$ and will move but if not then it will remain stationary.

$\therefore$ Horizontal component of P $= 14\cos 30 \approx 12.1 < F_{max}$

$\therefore$ Block will remain stationary.

P 7 1 4 0 3 A 0 1 0 2 8

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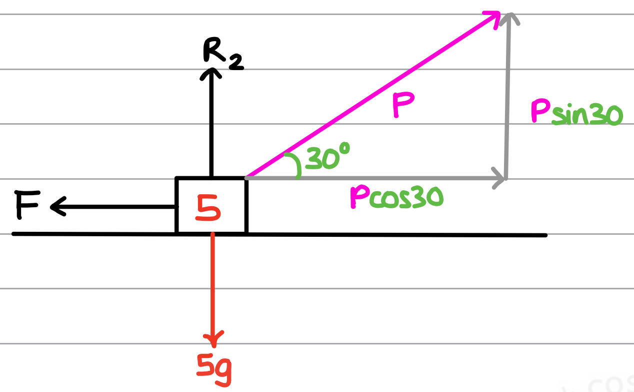
Question 4 continued

b) Draw a new diagram for the new unknown value of P.



Since its stated the block is on the point of sliping :

$P\cos 30 = F_{max}$

The block is still in vertical equilibrium ∴ $\boxed{\Sigma F = 0}$

∴ $R_2 + P\sin 30 - 5g = 0$ ∴ $R_2 = 5g - P\sin 30$

∴ Using the formula : $\boxed{F = \mu R}$,

$F_{max} = \frac{3}{7} \times (5g - P\sin 30)$

We can now set this equal to $P\cos 30$ and solve for P.

∴ $\frac{3}{7}(5g - P\sin 30) = P\cos 30$

∴ $\frac{15g}{7} - \frac{3}{7}P \times \frac{1}{2} = \frac{\sqrt{3}}{2}P$

∴ $\frac{\sqrt{3}}{2}P + \frac{3}{7}P \times \frac{1}{2} = \frac{15g}{7}$

∴ $P\left(\frac{\sqrt{3}}{2} + \frac{3}{14}\right) = \frac{15g}{7}$ ∴ $P = \dfrac{15g/7}{\frac{\sqrt{3}}{2} + \frac{3}{14}} = 19.43884465 \approx 19.4\text{ N}$ (3sf)

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Question 4 continued

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Question 4 continued

Q4

(Total 12 marks)

5.

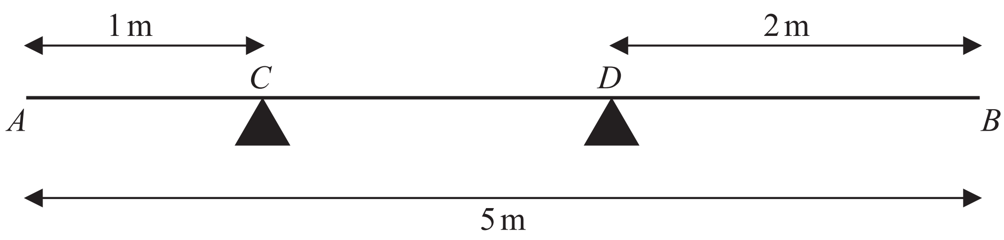


Figure 2

A uniform rod AB has length 5 m and mass 5 kg. The rod rests in equilibrium in a horizontal position on two supports C and D, where $AC = 1$ m and $DB = 2$ m, as shown in Figure 2.

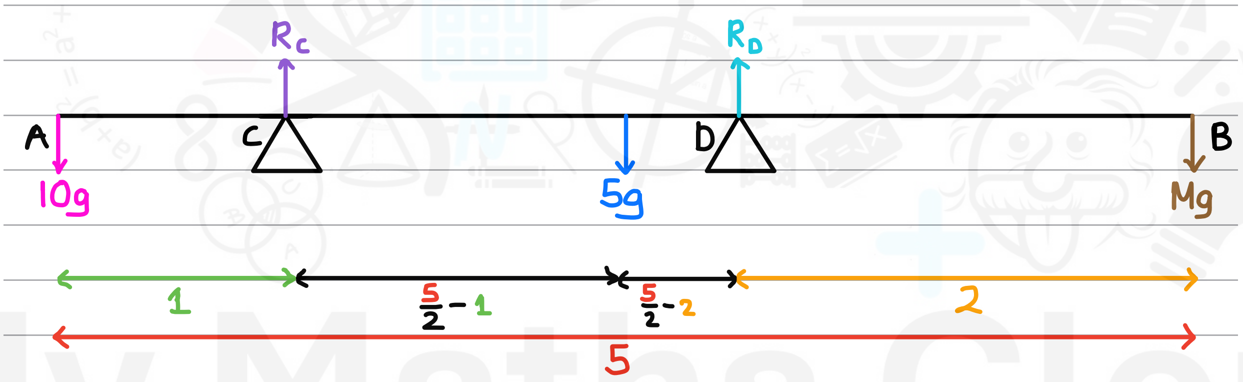
A particle of mass 10 kg is placed on the rod at A and a particle of mass M kg is placed on the rod at B. The rod remains horizontal and in equilibrium.

(a) Find, in terms of M, the magnitude of the reaction on the rod at C.
(3)

(b) Find, in terms of M, the magnitude of the reaction on the rod at D.
(3)

(c) Hence, or otherwise, find the range of possible values of M.
(3)

a) Draw a diagram labelling all the forces and lengths.



Since it's stated in the question that the beam is in equilibrium, the sum of the clockwise moments is equal to the sum of the anticlockwise moments, therefore :

$\boxed{\Sigma \text{ moments clockwise} = \Sigma \text{ moments anticlockwise}}$ where $\boxed{\text{moment} = \text{force} \times \text{perpendicular distance}}$

The clockwise forces are ones that go upwards from the left or downwards from the right of where moments are taken and anticlockwise forces are ones that go upwards from the right or downwards from the left of where moments are taken.

Question 5 continued

Taking moments around D to eliminate R_D :

$$\underbrace{Mg(2) + R_c(5-2-1)}_{\text{clockwise moments}} = \underbrace{5g\left(\frac{5}{2}-2\right) + 10g(5-2)}_{\text{anticlockwise moments}}$$

$\therefore 2Mg + 2R_c = \frac{5}{2}g + 30g$

$\therefore 2R_c = \frac{65g}{2} - 2Mg \qquad \therefore R_c = \frac{65g}{4} - Mg$

b) Taking moments around C to eliminate R_c :

$$\underbrace{Mg(5-1) + 5g\left(\frac{5}{2}-1\right)}_{\text{clockwise moments}} = \underbrace{R_D(5-2-1) + 10g(1)}_{\text{anticlockwise moments}}$$

$\therefore 4Mg + \frac{15g}{2} = 2R_D + 10g$

$\therefore 2R_D = 4Mg - \frac{5g}{2} \qquad \therefore R_D = 2Mg - \frac{5g}{4}$

c) We know that the reactions at C and D must be greater than zero as they can't be negative.

$\therefore R_D \geqslant 0 \qquad \therefore 2Mg - \frac{5g}{4} \geqslant 0 \qquad \therefore 2Mg \geqslant \frac{5g}{4} \qquad \therefore M \geqslant \frac{5}{8}$

$\therefore R_c \geqslant 0 \qquad \therefore \frac{65g}{4} - Mg \geqslant 0 \qquad \therefore Mg \leqslant \frac{65g}{4} \qquad \therefore M \leqslant \frac{65}{4}$

$\therefore \frac{5}{8} \leqslant M \leqslant \frac{65}{4}$

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Question 5 continued

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Question 5 continued

Q5

(Total 9 marks)

6. A particle P is moving with constant acceleration.

At time $t = 1$ second, P has velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$

At time $t = 4$ seconds, P has velocity $(5\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$

Find the speed of P at time $t = 3.5$ seconds.

(6)

It's stated that P moves with constant acceleration meaning SUVAT may be used.

We can set up SUVAT's for each senario to form equations

For time $t=1$

s -
u - u
v - $-\underline{i}+4\underline{j}$
a - a
t - 1

For time $t=4$

s -
u - u
v - $5\underline{i}-8\underline{j}$
a - a
t - 4

SUVAT formula without distance is :

$\boxed{v = u + at}$

$\therefore \begin{pmatrix}-1\\4\end{pmatrix} = u + a(1)$ ①, $\begin{pmatrix}5\\-8\end{pmatrix} = u + a(4)$ ②

We can now solve simultaneously.

$\therefore$ ② − ① $= u + 4a - u - a = \begin{pmatrix}5\\-8\end{pmatrix} - \begin{pmatrix}-1\\4\end{pmatrix}$ $\therefore 3a = \begin{pmatrix}6\\-12\end{pmatrix}$ $a = \begin{pmatrix}2\\-4\end{pmatrix}$

$\therefore$ Using equation ① : $u = \begin{pmatrix}-1\\4\end{pmatrix} - \begin{pmatrix}2\\-4\end{pmatrix} = \begin{pmatrix}-3\\8\end{pmatrix}$

Now a SUVAT can be set up for $t = 3.5$

s -
u - $\begin{pmatrix}-3\\8\end{pmatrix}$
v - v
a - $\begin{pmatrix}2\\-4\end{pmatrix}$
t - 3.5

$\therefore v = \begin{pmatrix}-3\\8\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix} \times 3.5 = \begin{pmatrix}4\\-6\end{pmatrix}$

$\therefore$ speed $= |v| = \sqrt{(4)^2 + (-6)^2} = 2\sqrt{13}\ \text{ms}^{-1}$ //

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Question 6 continued

(Total 6 marks)

Q6

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7. Two small children, Ajaz and Beth, are running a 100 m race along a straight horizontal track.

They both start from rest, leaving the start line at the same time.

Ajaz accelerates at $0.8\,\mathrm{m\,s^{-2}}$ up to a speed of $4\,\mathrm{m\,s^{-1}}$ and then maintains this speed until he crosses the finish line.

Beth accelerates at $1\,\mathrm{m\,s^{-2}}$ for T seconds and then maintains a constant speed until she crosses the finish line.

Ajaz and Beth cross the finish line at the same time.

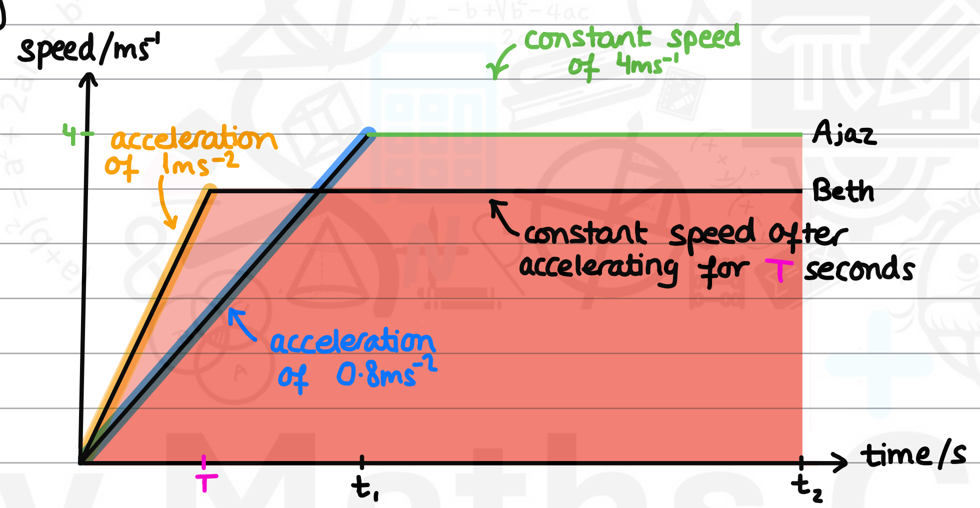
(a) Sketch, on the same axes, a speed-time graph for each child, from the instant when they leave the start line to the instant when they cross the finish line. **(3)**

(b) Find the time taken by Ajaz to complete the race. **(4)**

(c) Find the value of T **(4)**

(d) Find the difference in the speeds of the two children as they cross the finish line. **(2)**

a)



The distance travelled is equal to the area of the speed-time graph. This means that since Ajaz and Beth travelled the same 100m, the area under each of their lines should be equal hence Beth needs a lower final speed to make the areas equal.

b) We first need to find the length of time Ajaz accelerates for using the formula : $\boxed{a = \frac{\Delta v}{\Delta t}}$

$\therefore\ 0{\cdot}8 = \frac{4-0}{t_1-0} \qquad \therefore\ t_1 = \frac{4}{0{\cdot}8} = 5$ seconds

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$a = \frac{v}{t}$

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$v = at = T$

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Question 7 continued

Since the area under Ajaz's graph is equal to distance travelled :

$$100 = \overbrace{\frac{4 \times t_1}{2} + (t_2 - t_1) \times 4}^{\text{area under graph}}$$

$\therefore\ 100 = \frac{4 \times 5}{2} + 4t_2 - 4(5)$ $\quad\therefore\ 4t_2 = 100 + 20 - 10$

$t_2 = \frac{110}{4} = 27.5$ seconds

c) Speed of Beth after accelerating for T seconds can be found using : $\boxed{a = \frac{\Delta v}{\Delta t}}$.

$\therefore\ 1 = \frac{v - 0}{T - 0}$ $\quad\therefore\ v = T$

$\therefore$ Using the same fact in part b) .

$$100 = \frac{T^2}{2} + (t_2 - T)T$$

$\therefore\ 100 \times 2 = T^2 + 2t_2T - 2T^2$

$\therefore\ 200 = T^2 + 55T - 2T^2$ $\quad\therefore\ T^2 - 55T + 200 = 0$

$\therefore\ T = \frac{55 \pm 5\sqrt{89}}{2}$ However $T < t_2$ $\quad\therefore\ T = \frac{55 - 5\sqrt{89}}{2} \approx 3.92\text{s}$

d) Change in speeds $= 4 - v = 4 - T = 4 - 3.915.... \approx 0.0850\ \text{ms}^{-1}$ (3sf)

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Question 7 continued

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Question 7 continued

Q7

(Total 13 marks)

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two boats, P and Q, are moving with constant velocities.

The velocity of P is $15\mathbf{i}\,\mathrm{m\,s^{-1}}$ and the velocity of Q is $(20\mathbf{i} - 20\mathbf{j})\,\mathrm{m\,s^{-1}}$

(a) Find the direction in which Q is travelling, giving your answer as a bearing.

(2)

The boats are modelled as particles.

At time $t = 0$, P is at the origin O and Q is at the point with position vector $200\mathbf{j}$ m.

At time t seconds, the position vector of P is $\mathbf{p}$ m and the position vector of Q is $\mathbf{q}$ m.

(b) Show that

$$\overrightarrow{PQ} = [5t\mathbf{i} + (200 - 20t)\mathbf{j}]\,\mathrm{m}$$

(5)

(c) Find the bearing of P from Q when $t = 10$

(2)

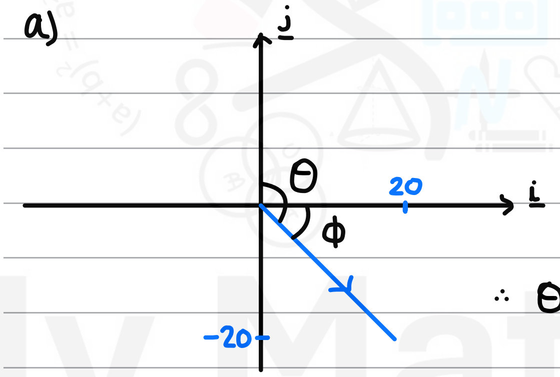
(d) Find the distance between P and Q when Q is north east of P

(5)

(e) Find the times when P and Q are 200 m apart.

(3)

a)



The direction of Q is θ where :

$\theta = 90 + \phi$

$\therefore \theta = 90 + \tan^{-1}\left(\frac{20}{20}\right) = 90 + 45 = 135°$

b) The position of a vector is given by : $\boxed{\underline{r} = \underline{r}_0 + vt}$, r_0 = initial position

$\therefore \underline{p} = (15i)t$, $\underline{q} = 200j + (20i - 20j)t$

Using the fact that : $\boxed{\overrightarrow{AB} = \underline{b} - \underline{a}}$ $\therefore \overrightarrow{PQ} = \underline{q} - \underline{p}$

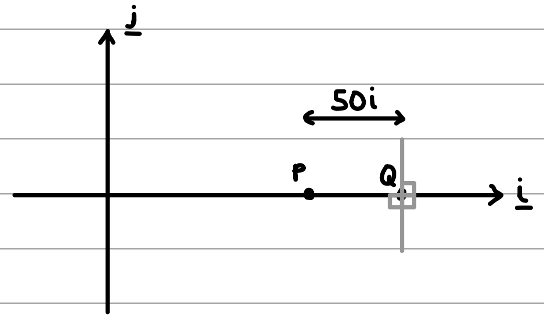
$\therefore \overrightarrow{PQ} = 200j + (20i - 20j)t - (15i)t$

$= (20t - 15t)i + (200 - 20t)j$

$= (5t)i + (200 - 20t)j$

Question 8 continued

c) $\overrightarrow{PQ}$ at $t = 10$: $(5(10))i + (200 - 20(10))j = 50i$



$\therefore$ Bearing of P from Q :

$= 90 + 90 + 90 = 270°$

since bearings go clockwise from north.

d) Q is north-east of P when $\overrightarrow{PQ} = xi + xj$ since both i and j components need to be equal.

$\therefore xi + xj = (5t)i + (200 - 20t)j$

$\therefore x = 5t$, $x = 200 - 20t$ Now we can find the time when Q is north-east of P.

$\therefore 5t = 200 - 20t$ $\therefore 25t = 200$ $\therefore t = \frac{200}{25} = 8$ seconds

$\therefore \overrightarrow{PQ} = (5 \times 8)i + (5 \times 8)j = 40i + 40j$

$\therefore$ Distance $= |\overrightarrow{PQ}| = \sqrt{(40)^2 + (40)^2} = 40\sqrt{2} \approx 56.6$m

e) The distance at any time is given by the modulus of $\overrightarrow{PQ}$.

$\therefore d_t = \sqrt{(5t)^2 + (200 - 20t)^2}$

$\therefore$ When $d = 200$, $200 = \sqrt{(5t)^2 + (200 - 20t)^2}$

$\therefore 200^2 = (5t)^2 + (200 - 20t)^2$

$\therefore 40000 = 25t^2 + 400t^2 - 8000t + 40000$

$\therefore 425t^2 - 8000t = 0$

$\therefore t = 0, \frac{320}{17}$ However $t \neq 0$ $\therefore t = \frac{320}{17} \approx 18.8$ seconds (3sf)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END